

Cylindrical Surface Pasting

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Abstract

In the paper, we present cylindrical surface pasting, an extension of standard surface pasting that uses the surface pasting technique to blend two surfaces. The major issues discussed here are the domain mappings and the mapping of the feature control points. There are two types of domain mappings, depending on whether we paste a cylinder on a NUBS sheet or another NUBS cylinder. The mapping of the feature control points has to address both continuity and shape issues.

1 Introduction

Hierarchical modeling is an important research topic. Many surfaces have varying levels of detail, and modeling techniques that explicitly represent these levels of detail are useful both in terms of reduced storage and in interactive modeling paradigms where users want to interact with their models at different levels of detail.

There are several methods for hierarchical modeling, including Hierarchical B-splines [6], various wavelet techniques, and LeSS [7]. Surface pasting is another hierarchical modeling method that has a couple of advantages over most other techniques. In particular, with surface pasting, the user can create a library of features, allowing for reuse of features. Further, unlike many techniques, the features, once pasted, can be reoriented in any direction on the base surface, and do not have to align with parametric directions.

Current surface pasting methods allow the user to paste one surface atop another. However, they do not allow for a single feature to connect two surfaces. Blending or filleting operations need to be employed to connect surfaces together. While there are many filleting methods, with the inspiration of standard surface pasting, we propose new a blending method in this paper, cylindrical pasting, that elaborates the domain mapping and displacement schemes of surface pasting, and applies it to place cylinders on NUBS base surfaces.

Our goal in cylindrical surface pasting is to extend the standard surface pasting method to a wider variety of modeling situations. Thus, while our method can be thought of as a blending method, we will treat it instead as a modeling technique, and in this paper we will focus on the mathematical details behind these operations rather than the user interface for modeling with these blends.

In the next section, we will state the relationship of cylindrical surface pasting to blending techniques. Then in Section 3, we will briefly review the standard surface pasting process. Section 4 is the heart of our paper, where we describe in detail the cylindrical surface pasting process. We conclude with some sample pasted surfaces and directions for future work.

2 Blending

Blending is an operation of creating smooth transitions between a pair of adjacent surfaces. Accordingly, the transition surface is simply called a *blend* or a *blending surface*. Blending methods that use parametric surfaces are the most popular techniques. Martin, Vida, and Varady have published a survey of different

blending methods using parametric surfaces that clarifies the nature of blending and the relationships between various parametric blending methods [10].

Using the Martin-Vida-Varady terminology, the cylindrical surface pasting method described in this paper can be thought of as a *local parametric-blending* method. In particular, we use a *trimline-based blend* as the basic idea for *Cylindrical Pasting*. In the following, a brief summary of the most important ideas in parametric blending is given. Figure 1 can be used as a guide to the different terms used in the blending literature.

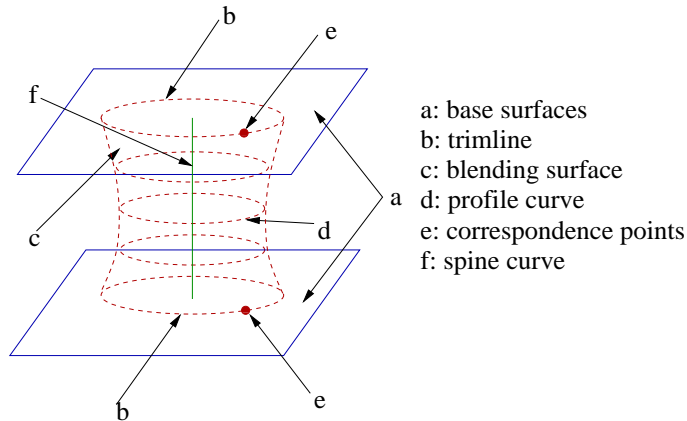


Figure 1: Terminology

The surfaces to be joined smoothly (the surfaces being blended) are called *base surfaces*. The curve that forms the common boundary of a base surface and the blend surface is called a *trimline*. The base surfaces are trimmed at these curves. In general, the blending surface is created as a surface or volume swept along a given longitudinal trajectory, which is called the *spine curve*. At each point of the spine, a cross-sectional *profile curve* is associated with it that locally defines the shape of the blend. A profile can be constant or varying along the spine, and can be symmetric or asymmetric, and can be defined as a circular or free-form arc. Having two trimlines, a corresponding point pair (one point from each trimline) can be joined by a profile curve. Correspondences between these pairs of points need to be established by the *assignment* process.

Cylindrical Pasting is similar to trimline-based methods, which are a class of techniques where an auxiliary spine is generated from the two trimlines, mainly for the purposes of assignment and the creation of profile curves. Since we know that blending replaces parts of the base surfaces with blending surfaces, one obvious way of specifying such an operation is to decide explicitly which parts are to be substituted by choosing where the trimlines should lie on the base surfaces. Once a pair of trimlines has been chosen, a spine curve is used to choose corresponding points on the trimlines to be assigned together. The final important phase of trimline-based methods is a method of generating profile information that makes it possible to define the profile curves that connect assigned pairs of trimline points and contribute to the blending surface.

3 Surface Pasting Process

Tensor product B-spline surfaces play an important role in current surface design [5], especially in surface pasting. Since standard surface pasting is the starting point for the technique described in this paper, we give a quick review of how it works. For details, see any of the earlier works on the subject [1, 2, 3].

The pasting process is illustrated in Figure 2. In surface pasting, we have both a base surface and a feature surface, each of which is in tensor-product B-spline form. The basic idea is to adjust the feature's control points in such a way that the boundary of the pasted feature lies on or near the base surface, and the shape of the pasted feature reflects the original shape of the feature as well as the shape of the base surface on to which it is pasted.

To map the feature's control points, we first embed the feature's domain in the feature's range (upper left of Figure 2); i.e., we make the feature's domain be a subspace of the feature's range. Typically, we construct the feature surface to allow for an embedding of the domain that places the boundary

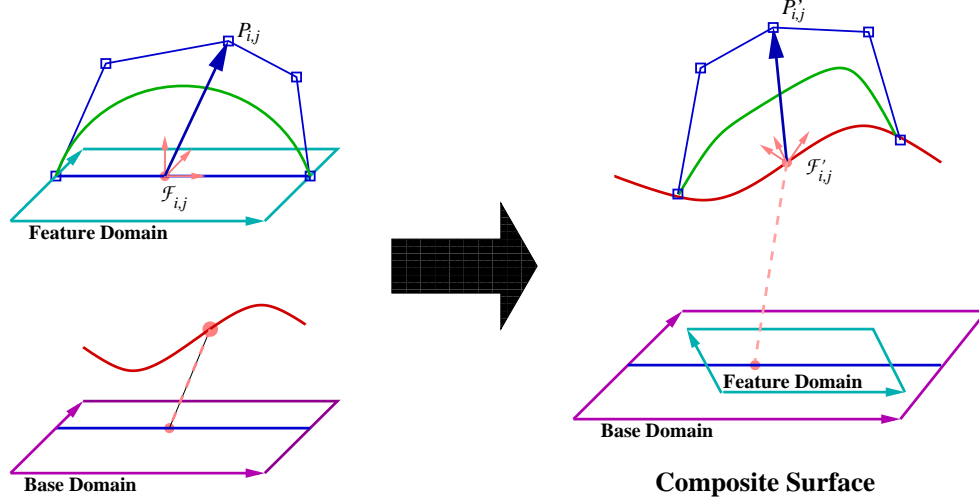


Figure 2: Pasting process

control points of the feature at the Greville points of the embedded domain. Next, we construct a local coordinate frame $\mathcal{F}_{i,j} = \{u_{i,j}, v_{i,j}, w_{i,j}, \mathcal{O}\}$ for each feature control point $P_{i,j}$ with the origin $\mathcal{O}_{i,j}$ of each frame being the Greville point corresponding to the control point, with two of the frame's basis vectors being the parametric domain directions and the third basis vector being the direction perpendicular to the domain. Each control point $P_{i,j}$ is then expressed relative to its local coordinate frame $\mathcal{F}_{i,j}$ as $P_{i,j} = \alpha_{i,j}u_{i,j} + \beta_{i,j}v_{i,j} + \gamma_{i,j}w_{i,j} + \mathcal{O}_{i,j}$.

Next, we associate the feature's domain in the base's domain (right half of Figure 2). This gives us the location on the base surface on to which we want to locate the feature. We now map each coordinate frame $\mathcal{F}_{i,j}$ on to the base surface, giving a new coordinate frame $\mathcal{F}'_{i,j} = \{u'_{i,j}, v'_{i,j}, w'_{i,j}, \mathcal{O}'_{i,j}\}$, whose origin $\mathcal{O}'_{i,j}$ is the evaluation of the base surface at $\mathcal{O}_{i,j}$, and two of its basis vectors lie in the tangent plane of the base surface at that point, the third being perpendicular to the tangent plane. We then use the coordinates of each feature control point $P_{i,j}$ relative to $\mathcal{F}_{i,j}$ to weight the elements of the frame $\mathcal{F}'_{i,j}$. This gives us the location of the pasted feature control point, $P'_{i,j} = \alpha_{i,j}u'_{i,j} + \beta_{i,j}v'_{i,j} + \gamma_{i,j}w'_{i,j} + \mathcal{O}'_{i,j}$.

4 Cylindrical Surface Pasting

Cylindrical Pasting is a surface modeling tool that integrates the techniques of parametric trimline-based blending into surface pasting, and creates a smooth transition cylinder between two base surfaces. We made modifications to the major pasting techniques (domain mapping and control points displacement) to adapt standard surface pasting to the cylindrical pasting environment.

In this paper, we give details of the final technique that we used for cylindrical surface pasting. A more complete description and a description of other ideas we tried that did not work can be found in a technical report [9]. However, we will describe one approach that we discarded and replaced with a better method, since this improved technique works both in standard pasting and cylindrical pasting.

The main idea in surface pasting is the mapping of the feature control points to get the feature surface to lie in the appropriate place relative to the base surface. There are three types of control points to map, each of which requires a different mapping technique. The first control points are those along the boundary of the feature. These should be mapped to achieve approximate C^0 continuity. The second layer of control points are mapped to achieve approximate C^1 continuity. And the remaining interior control points are mapped to achieve the desired feature shape. In this paper, we focus on the mapping of the first two layers, as their mapping is the pasting process; for completeness, we also give the mapping of the remaining control points, although they could be mapped using any standard extrusion method.

We will begin by stating the representation of the cylindrical feature used in our system. Next, we describe the first step to mapping the feature boundary control points, which is to associate the feature domain with the base domain. We then give the mapping of the first and second layers of control points. We then discuss our mapping of the remaining interior points. In Section 5, we give a brief overview of

our user interface, and show some results of the cylindrical pasting process.

4.1 Representing a Cylinder Using NUBS

Our work deals with non-uniform B-splines (NUBS). To represent a cylindrical shape using a NUBS surface, we use the standard trick of identifying one of the edges of the domain rectangle with the opposite edge of the domain rectangle. The cross section of a cylinder is a circular curve. Although circles cannot be represented exactly using a NUBS curve, a NUBS curve can represent a closed curve that is a good approximation a circle [4], although for our implementation we constructed the circular approximation by hand. To represent a closed curve with a cubic NUBS, we set the last three control points to be the first three control points, with an appropriate setting of the knot vector.

Mathematically, if we have a cubic B-spline with a knot vector $\{v_0, \dots, v_N\}$ and control points P_0, \dots, P_{N-3} , the following conditions must hold to get a closed curve:

$$P_0 = P_{N-5}, \quad P_1 = P_{N-4}, \quad P_2 = P_{N-3}, \tag{1}$$

and

$$\begin{aligned} v_1 - v_0 &= v_{N-3} - v_{N-4}, & v_2 - v_1 &= v_{N-2} - v_{N-3}, \\ v_3 - v_2 &= v_{N-1} - v_{N-2}, & v_4 - v_3 &= v_N - v_{N-1}. \end{aligned} \tag{2}$$

(Note: we are using the knot vector typically used with the blossoming variant of the B-spline; other forms of B-splines will typically put an additional knot at each end of the knot vector.)

A tensor B-spline surface has a two-dimensional domain defined in two parametric directions, U and V . We represent our cylinders by a rectangular domain where the V direction joins itself as in Equations 1 and 2, and U aligns with the axis of the cylinder. We will use a knot vector with full end knot multiplicity in the U direction.

4.2 Mapping of Domain for Cylinders

The first step in performing standard surface pasting is to map the domain of the feature surface to the domain of the base surface. The same procedure has to be done for Cylindrical Pasting. The rectangular domain of the feature cylinder has to be first mapped inside the domain of the base surface.

A cylinder can be pasted on two types of NUBS base surfaces: a normal NUBS surface, or a cylindrical NUBS surface. Depending on the type of the base surface, the rectangular domain of the feature cylinder will be transformed to the base domain in two different ways.

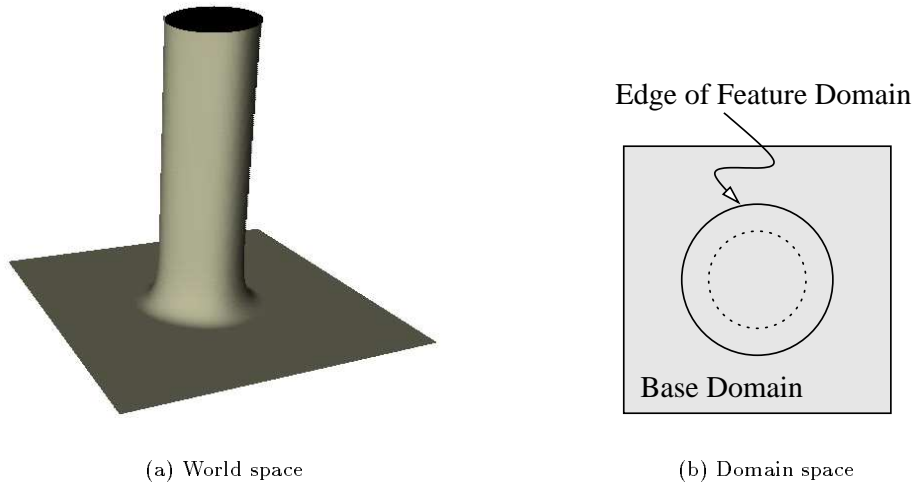


Figure 3: A blending cylinder on a normal NUBS surface.

In the first case, the base surface is a normal NUBS surface with a rectangular domain. Only one of the two edges of the feature cylinder will be pasted on the base, as shown in Figure 3(a). We locate

the position of the edge of the feature on the base surface through a domain association. The edge of the feature domain corresponding to the edge of the feature surface that is to lie on the base surface is mapped to circle in the base domain as shown in Figure 3(b). By default, we initially locate the domain for the feature cylinder at the center of the base domain with a predefined radius; the user may scale and translate this circle within the base domain. The second circle (dotted) in this figure is used for mapping the derivatives, as discussed in the next section.

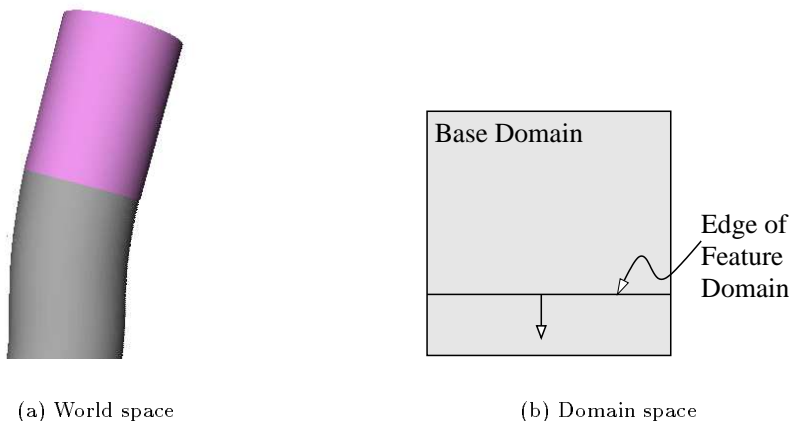


Figure 4: A blending cylinder on a cylindrical NUBS surface.

In the second case, both the base and features surfaces are NUBS cylinders. Again, only one of the feature cylinder’s edges is pasted on the base as illustrated in Figure 4(a), with the top cylinder as the base. To locate the edge of the feature surface on the base surface, we again map an edge of the feature’s domain into the base domain. As shown in Figure 4(b), the mapping of this edge is different. Since the base is a cylinder, we map the edge of the domain to a line that spans the base domain. Since the two sides of the base domain represent the seam of the cylinder, we have mapped the closed curve of the edge of the feature surface to a closed curve on the base surface. The arrow in this figure is used to map the derivatives, as discussed in the next section.

4.3 Control Points Displacement Scheme

As the mechanism of Cylindrical Pasting is based on the idea of trimline-based blending, the major issue is to determine the trimlines on the base surfaces. The body of the blend is constructed from the spine curve to be defined from the trimlines, and the profile curves along the spine curve.

In standard pasting, each control point of the feature surface is represented relative to a local coordinate frame. These frames are then mapped on to the base surface, and each feature control point is placed relative to the image of its local frame. For cylindrical pasting, we only have an identification of the first two rows of the feature control points with the base surface. These two layers can be mapped using the standard pasting method, although we will present a better mapping of the second layer of control points. The remaining feature control points must be mapped in a different manner. In this section, we discuss the mapping of the first two layers of control points, and in the next section we discuss the mapping of the remaining layers of control points.

The first two layers of control points at either end of the cylinder determine the position and first derivatives along the boundary. We call these two rows of control points the 0^{th} and 1^{st} layers in the world space, denoted as L_0 and L_1 , respectively. L_0 is analogous to the trimline in the trimline-based blending method.

We mapped the L_0 layer of the feature cylinder in the same manner as standard pasting. Each L_0 point is located at the Greville point in the embedded feature domain, giving a zero displacement vector relative to its local frame, and the standard pasting procedure is used to paste these points. In other words, the boundary control points of the feature cylinder map to points on the base surface. This procedure is done for both edges of the feature cylinder. The result is that the boundary of the pasted

cylinder will lie close, but not directly on, the base surface. If the C^0 discontinuity is too high, it can be reduced by performing knot insertion on the feature cylinder, as is done for standard pasting.

Next, we map the L_1 layer to approximate C^1 continuity, which in standard pasting required the displacement vectors for the L_1 layer control points to be set to zero. With cylindrical pasting, we only have the boundaries of the cylinder’s domain associated with the base domain. So in our initial attempt, for pasting a cylinder on a NUBS surface we placed a second circle in the base domain, with the same center but smaller radius than the first circle (the dotted circle of Figure 3(b)). And to paste a cylinder on another cylinder, we associated the L_1 layer with the base domain by using a vector perpendicular to the location of the feature domain edge within the base domain (the arrow in Figure 4(b)). In both cases, once the identification of the L_1 layer of the feature with the base domain was made, we mapped the L_1 layer of the feature in the same manner as standard pasting.

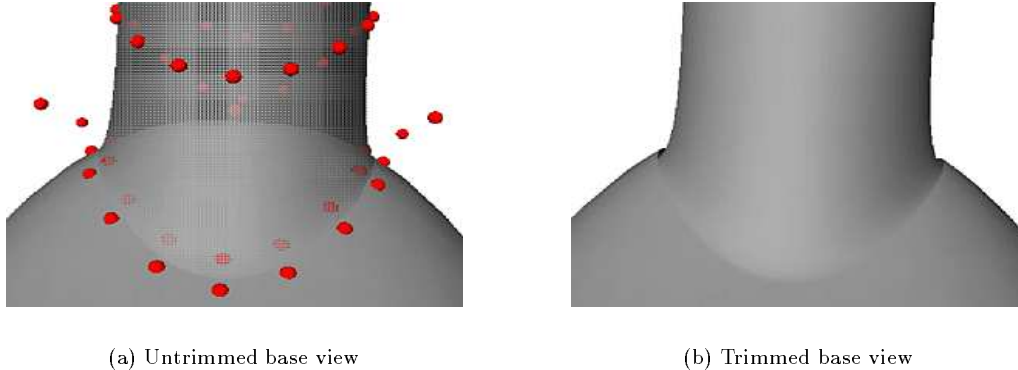


Figure 5: Pasting with C^1 continuity.

To get a better feel for the C^0 and C^1 discontinuities, we looked at both the trimmed and untrimmed surfaces, and discovered two problems. The first problem is that while the boundary of the feature lies near the base, it is still unacceptably far from it as seen in Figure 5(b). This problem can be handled by inserting more knots in the V parametric direction. However, using the standard pasting method for the second layer of control points was problematic: the C^1 discontinuity was still unacceptably high Figure 5(a) (in this figure, the feature cylinder is rendered as partially transparent). Adding knots in the V parametric direction has no effect on this discontinuity, and adding knots in the U parametric direction yields only small improvements.

To reduce the C^1 discontinuity, we changed the method for pasting the L_1 layer of control points. For cylindrical pasting, we still paste the L_0 layer of control points by using the method of standard pasting using zero length displacement vectors. Then, at each feature control point $C_{0,j}$ of L_0 , we construct a coordinate frame $\mathcal{F}_{0,j}$, where $C_{0,j}$ is the origin, the unit derivative vector in the V direction is one coordinate vector, the derivative vector in the U direction is the second coordinate vector, with their cross-product forming the third derivative vector (Figure 6(a)).

Next, we map the frames for the L_0 layer to the embedded domain of the base surface (Figure 6(b)). Here, the two frame vectors that are tangent to the unpasted feature surface are mapped to the domain plane so that the basis vector tangent to the circle along the edge of the cylinder maps to be tangent to the circle in the base domain, and other tangent vector that lies in the tangent plane of the cylinder maps to be perpendicular to the circle, pointing inside the circle (the third basis vector is mapped parallel to the z -axis). We then map the frames on to the base surface and construct the $\mathcal{F}'_{0,j}$ frames. Each L_1 layer control point $P_{1,j}$ is then expressed as a displacement relative to frame $\mathcal{F}_{0,j}$, and as with standard pasting these values are used to weight the elements of $\mathcal{F}'_{0,j}$ to get the location of $P'_{1,j}$.

The net effect of the new method is to map differences of control points on the L_1 and L_0 layers (e.g., $P_{1,j} - P_{0,j}$) to cross-boundary derivatives of the base surface. With the new scheme both the C_0 and C_1 discontinuities are decreased as we insert knots in the V parametric direction of the feature.

This new method of mapping the L_1 layer has a lower C^1 discontinuity than the original method for mapping this layer as can be seen in the other images in this paper. Although devised for cylindrical pasting, this method for mapping the second layer of control points could easily be incorporated into standard pasting, and should give a reduction in C^1 discontinuity with no increase in computational cost.

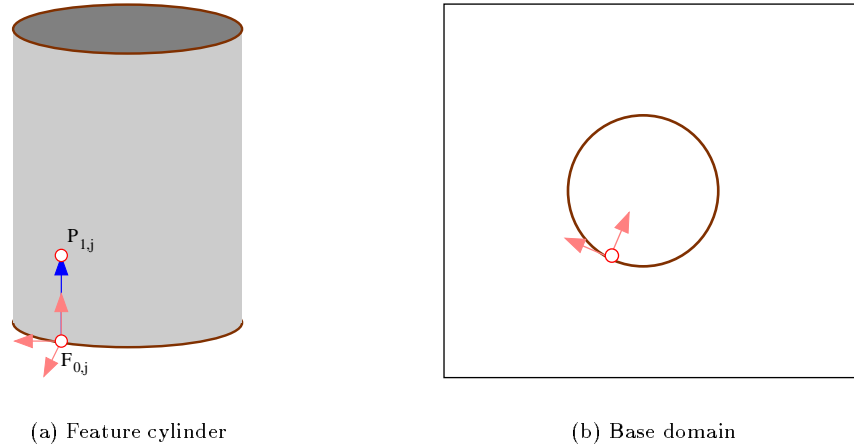


Figure 6: Cylindrical displacement mapping

4.4 The Remaining Control Points

Once we have mapped the L_0 and L_1 layers of both ends of the feature on to two bases surfaces, we need to map the remaining layers of the feature's control points. This is a standard extrusion problem; since our focus was on the pasting process (i.e., the mapping of the boundary control points), we implemented a simple technique using a spine curve, reparameterized to a near arc length parameterization, and then mapped the L_1 layers along this curve.

We considered using cubic Hermite splines to connect the L_i layers as Kim and Elber did [8]. Had we done this, then the mapping of the L_0 and L_1 layers described in the previous section would complete the mapping of our cylinder, and our method would essentially be identical to that of Kim and Elber. However, we intend to use our method for both blending and for longer connecting pieces, and we found that using only four layers of control points gave poor results for longer connecting pieces.

If we have more than four layers of control points, after mapping both pairs of L_0 and L_1 layers, we need to map the remaining interior control points of the feature cylinder. Initially we tried some simple linear interpolation techniques of the L_1 layers to locate the remaining interior control points. However, we found that these techniques gave us sharp creases and/or skews in our connecting cylinder, as illustrated in Figure 7.

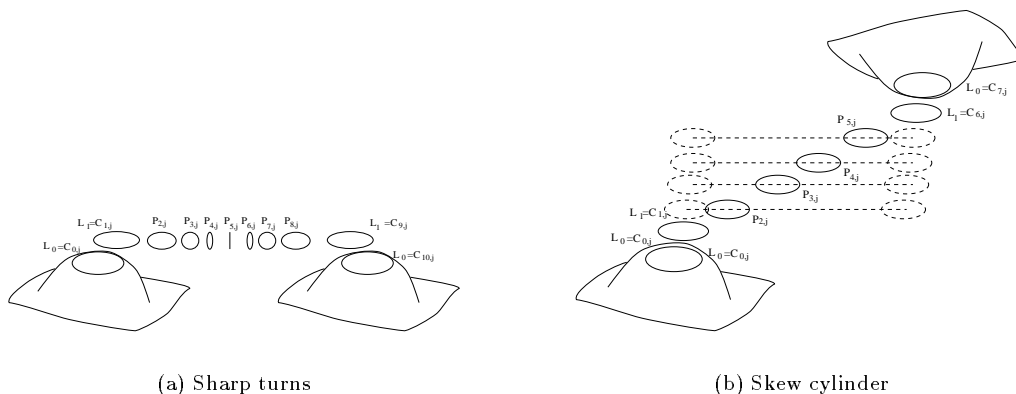


Figure 7: Linear interpolation leads to poor blends.

Instead, we decided to use a spine curve to specify the approximate path of the feature cylinder, and construct the remaining interior feature cylinder control points by mapping the L_1 layers to lie roughly

perpendicular to this spine curve. The rest of this section gives the details of the construction of this spine curve and the mapping of the L_1 layers.

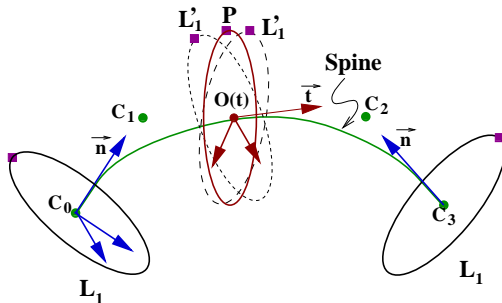


Figure 8: Method to determine an inbetween layer of control points.

To get a well-shaped blending cylinder, we constructed the interior control points around a spine curve. This spine curve plays the role of the skeleton for the cylinder. It is a simple cubic Bézier curve defined by four control points: C_0 , C_1 , C_2 and C_3 . Each of the two end points, C_0 and C_3 , is the average point of the corresponding L_1 layers of control points. We then construct vectors \vec{n}_0 and \vec{n}_3 at C_0 and C_3 by summing the crossproducts with the surrounding points in the layer:

$$\begin{aligned} \vec{n}_0 &= \sum_j (C_{1,j} - C_0) \times (C_{1,j+1} - C_0) \\ \vec{n}_3 &= \sum_j (C_{n-1,j} - C_3) \times (C_{n-1,j+1} - C_3) \end{aligned}$$

where $j + 1$ is taken modulo $m + 1$, with $n + 1$ and $m + 1$ being the number of control points in the two parametric directions. The orientations of the \vec{n}_i are set to point away from the corresponding base surfaces.

Once we have C_0 , C_3 , \vec{n}_0 and \vec{n}_3 , we locate the other two control points, C_1 and C_2 by placing C_1 on the line given by C_0 and \vec{n}_0 , and C_2 on the line given by C_3 and \vec{n}_3 . The C_i are now the control points of a simple cubic Bézier curve as shown in Figure 8. The distance along the \vec{n}_i that we place C_1 and C_2 is a curvature parameter that determines the bending of the cylindrical body, and is made available as a shape parameter for the user.

To define the cylindrical body, each remaining layer of control points P is built as a ring of points along the spine. We will build each of these curves as a linear interpolation of the mapped images of the two L_1 layers at both ends. These mapped images are denoted as L_1' in Figure 8.

The effect we want is for the left L_1 layer to sweep along the curve, gradually transforming into the right L_1 . If we have $n + 4$ layers of control points in our blending cylinder, then four layers are L_i layers, and we need n profile curves. To construct the L_1' and P layers, we first build a local coordinate frame at C_0 and one at C_3 , and represent each of the control points in the L_1 layers relative to the corresponding frame. We then map these coordinate frames along the curve so that they are centered at a point $O(t)$ on the spine obtained by evaluating $O(t) = \sum C_i B_i^3(t)$ at some set of t values; the mapping of the basis vectors of the frames is described below.

While the values $t = \frac{i}{n+1}$, for $i = 1 \dots n$, might seem like appropriate choices for sampling O , this Bézier curve is not arc-length parameterized. Thus, with uniform samplings of t , we get non-uniform samples on O , resulting in a blend surface with twists. To address that problem, we made an approximate arc-length parameterization of O by sampling O uniformly, computing the distance between these sample points, and using these distances to reparameterize the curve. The result is a close-to-arc-length parameterization, and rings of control points that are uniformly spaced over O .

Once we have the sample points on O , we need to map the L_1 layers to these sample points. We initially considered the idea of rotating L_1 along the spine curve with progressive degrees to get mapped images L_1' has been considered. Unfortunately, it is unclear how to find the appropriate degree variations for how much each L_1 should rotate to give the final profile to best represent the geometry of its base.

Instead, we used a geometric transformation of \vec{n} , mapping \vec{n} to the vector \vec{t} tangent to the spine curve at $O(t)$. This gives the direction for locating the mapped coordinate frame derived from C_0 , hence, the mapped control points can be used to locate L'_1 . Applying the same process to L_1 at C_3 , two mapped curves L'_1 are obtained at $O(t)$. To obtain the final profile curve P that reflects the transition between the base surfaces, we applied linear interpolation (in the layer number) on the generated L'_1 s.

4.5 Correspondence Assignment Process

The cylindrical pasting process constructs the first two layers of control points on either end of the pasted cylinder by associating boundary control points with two edges of the cylinder domain, and then mapping these two edges into the base domain. The interior control points are found by transforming the second layer at either end of the cylinder along a curve, and then blending corresponding points, as described earlier in this chapter. However, one question that we have left unanswered is how to set up a correspondence between the two layers of blended control points.

The correspondence process is a non-trivial one. If we make a poor match between the two layers, then we introduce a twist in our blending surface. In our prototype for cylindrical pasting, we used the following process (which we note is inadequate in general):

1. Using the Cartesian coordinate system for the range space and the normal to the plane approximating the each second layer of pasted control points, find the coordinate direction most perpendicular to each normal.
2. Select the control point within the layer whose coordinate relative to the selected axis is a maximum.
3. Repeat the selection process with the second layer at the other end of the pasted cylinder.
4. Associate the two selected control points, and associate the remaining control points within the layers starting from the selected control points.

As stated, the last step still leaves unspecified the direction around the ring of control points. We chose this direction arbitrarily, and allow the user to change the direction for either ring of control points.

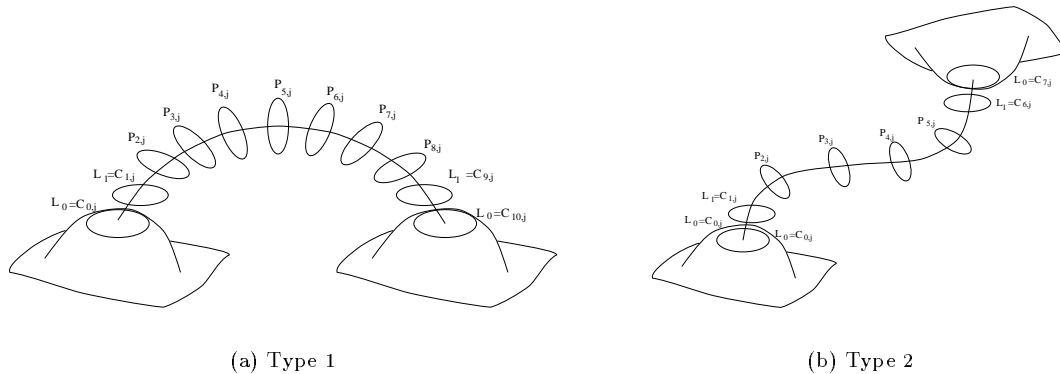


Figure 9: Good blend.

Note that this method works reasonably well if the two layers have relative locations similar to that on the right of Figure 9, but is a poor choice if the layers have relative locations similar to that on the left of Figure 9. Using a better method for solving the correspondence problem has been left for future work.

For more details on our method, see the technical report [9]; see the Vida-Martin-Varady survey [10] for references to other extrusion methods.

5 Results

We tested our cylindrical pasting method by blending two surfaces. Two examples were shown earlier (Figures 3 and 4). A third example is shown in Figure 10. In this figure, the bottom surface is a plane,

while the top surface is a curved surface. The plane provides a useful test case since the pasting method for the boundary control points will result in the boundary of the feature meeting the plane with C^1 continuity. Note, however, that once we trim the base, we will not have a C^0 join, since the feature boundary is not the trim curve. In any case, in this image we see that cylindrical pasting has the desired effect.

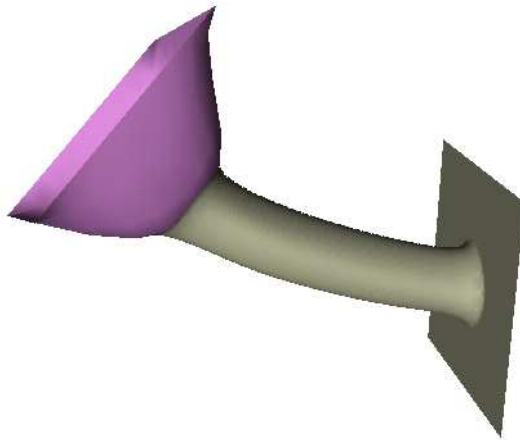


Figure 10: Cylindrical pasting example.

Our system was designed to test the mathematical ideas, and was implemented with a simple user interface. The following is an overview of the system. The user selects two surfaces to be joined with the mouse. The system places the boundary of the feature's domain in each of the base domains. The L_0 and L_1 layers of the feature's control points are mapped on to the base, an initial spine curve is created, and the remaining layers are set using the method described in Section 4.4.

The user can adjust both domains using sliders to adjust the radius of the circles, and can drag the circles/lines representing the feature domains in a pop-up domain window. The user also can adjust the spine curve using sliders to adjust the curvature of the spine curve. In the current system, the user has to visually inspect the joins of the features to the base, and tell the system to insert knots in the feature if the discontinuities are too high. After any adjustment, the system recomputes the blending feature.

Using our surface editor, we were able to drag the ends of the cylinder along the base surfaces and adjust other parameters at interactive speeds. The C^0 discontinuity was only visible when a small number of control points were used in the V parametric direction. This gap would disappear after performing knot insertion in the V direction, although some pixel drop-out was still visible due to the mismatch in tessellations between the base and feature surfaces.

The C^1 discontinuity was not visible, although if the cross-boundary tangents are too short then the sharp curvature at the join was visible.

6 Conclusions and Future Work

The goal of this work was to extend the surface pasting method to allow us to use the feature surfaces to connect two base surfaces. The results in this paper show how to modify the pasting process to achieve this goal. We can now use the surface pasting method to connect two surfaces, similar to other blending schemes. The advantages of cylindrical surface pasting are low degree connecting pieces that can be modified at interactive speeds.

Further, we found an improvement to the standard pasting method for mapping the cross-boundary control points. Our new method of mapping these control points results in a lower C^1 discontinuity, and knot insertion in the V direction reduces both the C^0 and C^1 discontinuities (for standard surface pasting, knot insertion in the V direction only reduces the C^0 discontinuity).

At this point, several major issues still need to be explored:

1. Domain curves. We used a NUBS approximation to a circle to map the edge of the feature domain into the base domain. However, we have no guarantee that a circle in the base domain will map to a nice curve on the base surface. An ideal user interface would either allow the user to specify a

curve on the base surface, or the system would find automatically a good first guess of a curve on the base surface, and map this curve backwards into the base domain.

2. Hierarchical Modeling. The goal of surface pasting is to provide a hierarchical modeling method that allows reuse of feature surfaces. The current version of cylindrical surface pasting is non-hierarchical. While some aspects of extending cylindrical surface pasting to be a hierarchical method are straightforward, other aspects will be more difficult. In particular, if you paste both ends of a cylinder on to the same surface, then the resulting surface will be of higher genus than the original base surface.¹ Such topological issues will complicate the hierarchical cylindrical surface pasting technique.

Recently, Gozonlez-Ochoa and Peters [7] have developed an offset method similar to surface pasting. Their method works on top of a winged-edge data structure, and readily solves these topology issues. Hierarchical modeling with cylindrical surface pasting will probably need to take a similar approach.

3. Fine tuning. The current system was a proof-of-concept implementation. The user interface is low level, with the user directly adjusting various parameters through sliders. Further, parts of what is now directly controlled by the user could be automated, such as automatically inserting knots to reduce the C^1 and C^0 discontinuities to the user specified tolerance.

Finally, our construction of the interior control points was an ad hoc construction intended to test the feasibility of cylindrical surface pasting. Instead, the shape of the cylindrical blend could be automatically set to achieve various goals (closest fit to a cylinder, minimize maximal curvature, etc.).

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¹ Thanks to Jorg Peters for pointing this out.