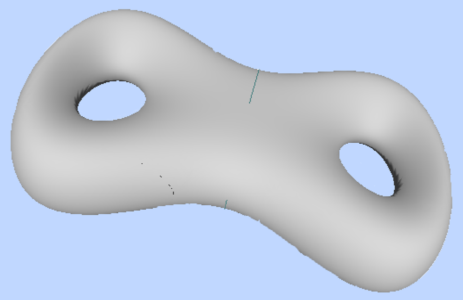
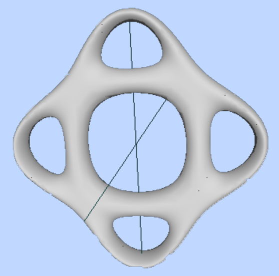
Extending the A-Patch single sheet conditions to enable the tessellation of algebraics



http://www.cs.uwaterloo.ca/research/tr/2009/CS-2009-21.pdf



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Abstract

A-Patches are a form of representation of an algebraic curve or surface over a simplex. The A-Patch conditions can be used as the basis for an adaptive subdivision style marching tetrahedra algorithm whose advantage is that it guarantees that we do not miss features of the algebraic: singularities are localized, and in regions around nearby multiple sheets, the subdivision process continues until the sheets are separated.

Unfortunately, the A-Patch single sheet conditions are too strict: for some algebraics, the subdivision process converges slowly or fails to converge. In this poster, I give an additional single sheet condition for curves that allows for convergence of this process. I also give additional conditions for surfaces that trades off some of the single sheet guarantees for improved convergence.

A-Patches

Trivariate Bernstein polynomials:

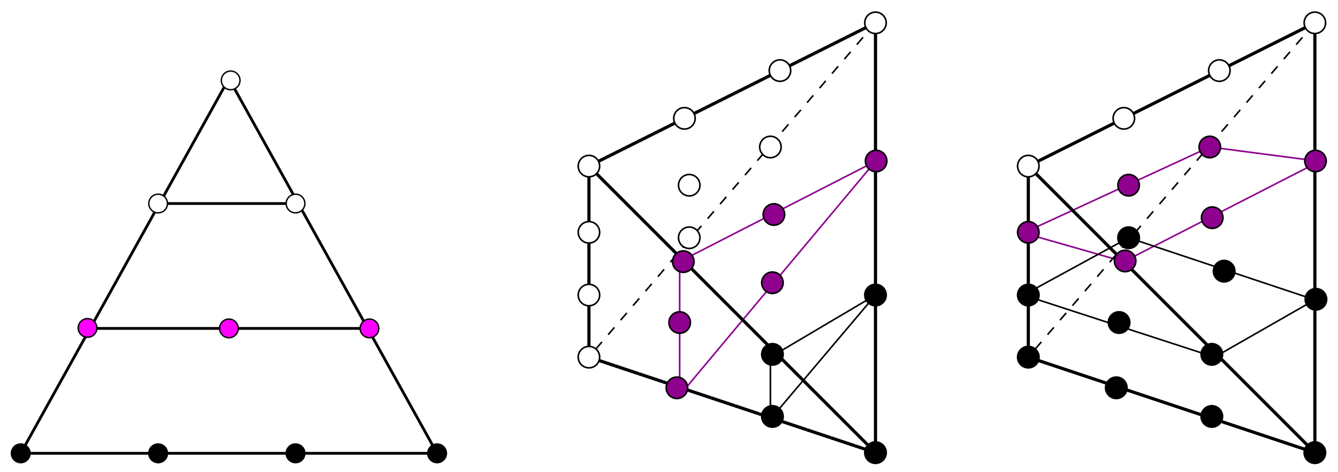
$$B^n_{\vec{i}}(P) = \binom{n}{\vec{i}} p_0^{i_0} p_1^{i_1} p_2^{i_2} p_3^{i_3},$$

where $\vec{i} = (i_0, i_1, i_2, i_3)$ with $i_0, i_1, i_2, i_3 \le n$ and $i_0 + i_1 + i_2 + i_3 = n$, and where (p_0, p_1, p_2, p_3) are the Barycentric coordinates.

An A-Patch weighs scalar values with the Bernstein basis:

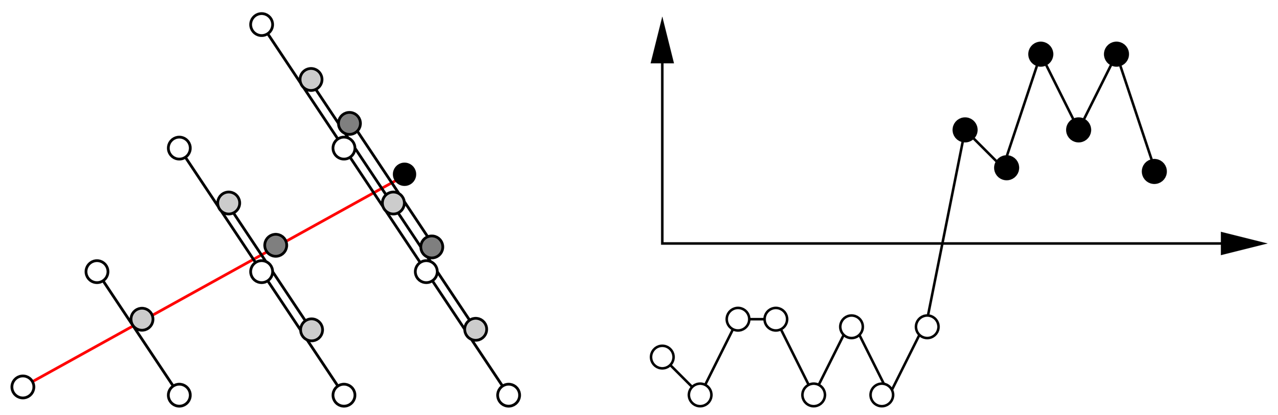
$$F(P) = \sum_{\vec{i}} c_i B^n_{\vec{i}}(P).$$

A-Patch coefficients are separated by sign with an intermediate layer of mixed signs:



Separation layers for curves and for 3- and 4-sided surface A-Patches

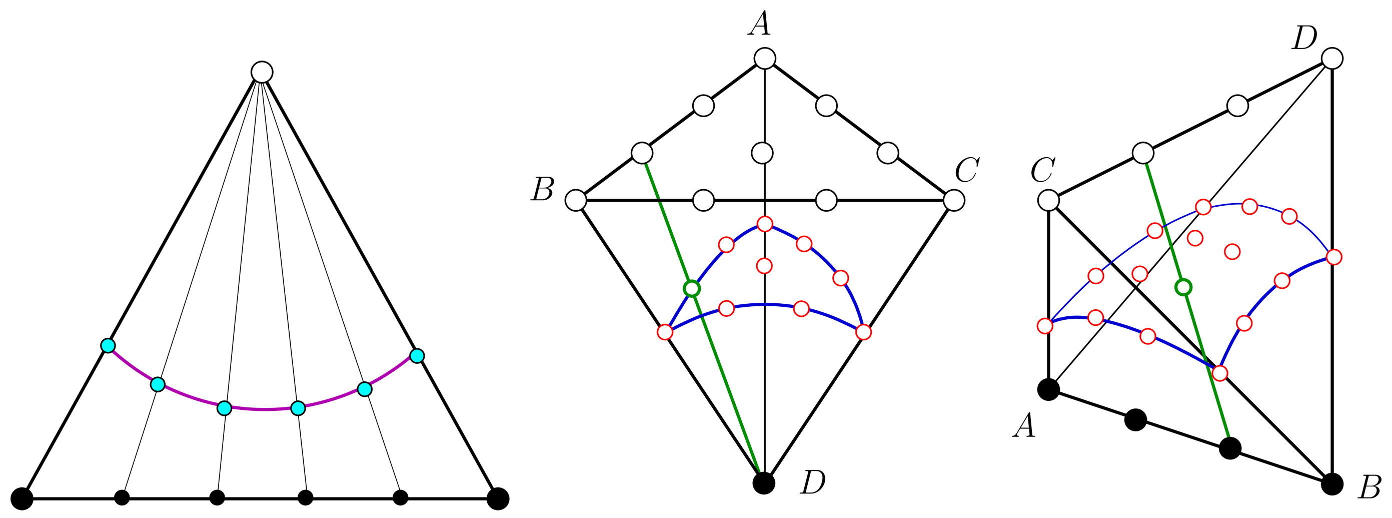
- Separation guarantees that a single sheet of the algebraic surface passes through the tetrahedron:



Evaluating along line from point to opposite edge results in sequence of coefficients with one sign change; by the variation diminishing property, there is exactly one zero.

Single sheet give means of tessellating

Root find along lines from corner to opposite edge/face (if curve or 3-sided A-Patch) or edge to opposite edge (if 4-sided A-Patch)

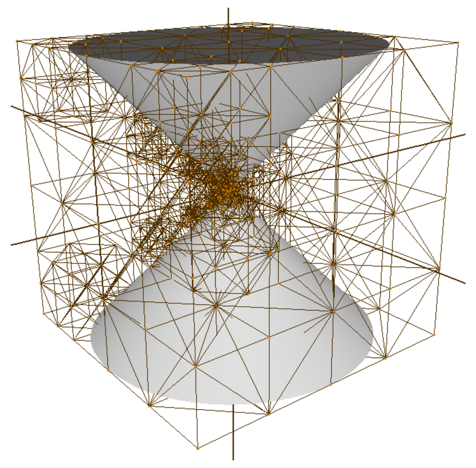


Plotting the curve/surface from an A-Patch

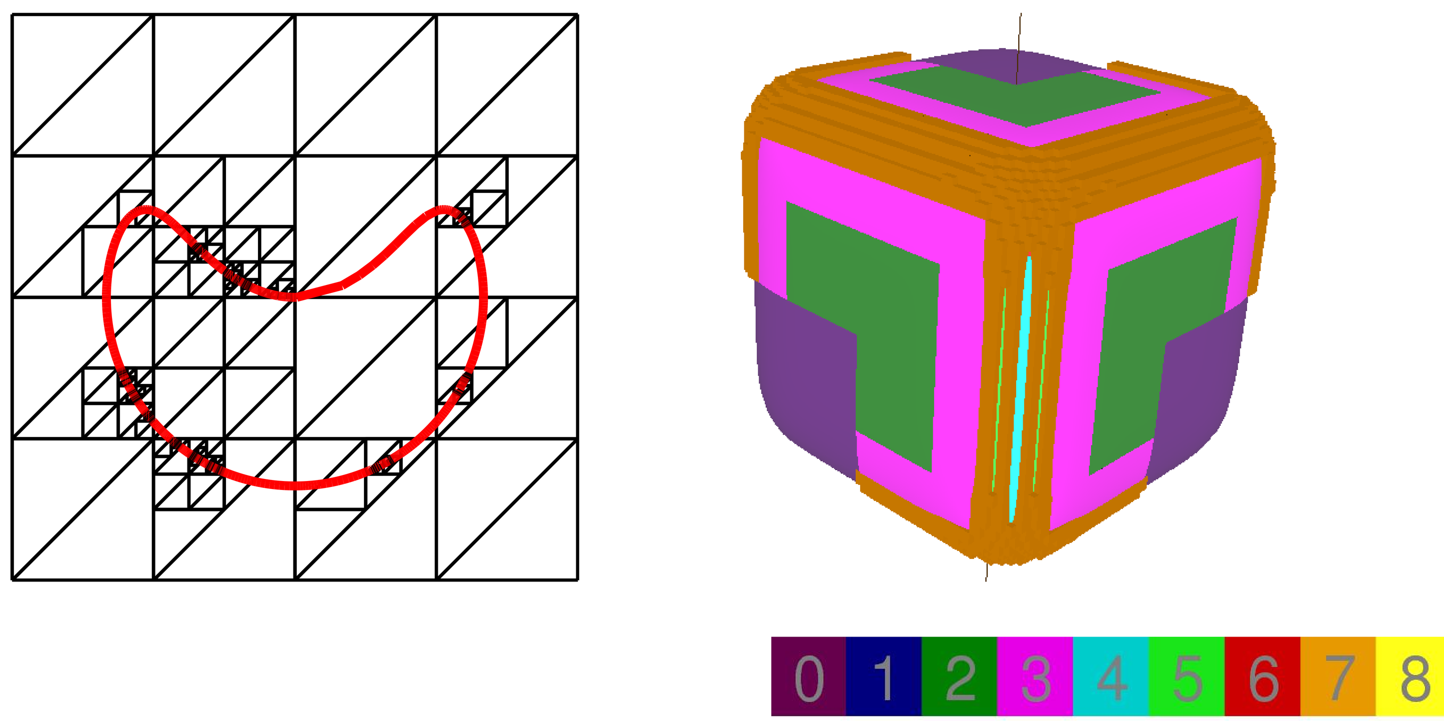
Adaptive Tessellation

These observations lead to the following algorithm for tessellating algebraic curves and surfaces:

- Subdivide space into triangles/tetrahedron
- Convert to Bernstein form of algebraic within each simplex
- For each simplex, test the Bernstein coefficients:
  - All patches have all positive/negative coefficients: surface does not pass through the area
  - All patches are in A-Patch format: tessellate
  - Any patch has mixed coefficients: subdivide



Problem: Sometimes subdivision doesn't converge



Left: Clown Smile fails to converge; Right: Cube fails to converge (color indicates subdivision depth).

Extended single-sheet conditions

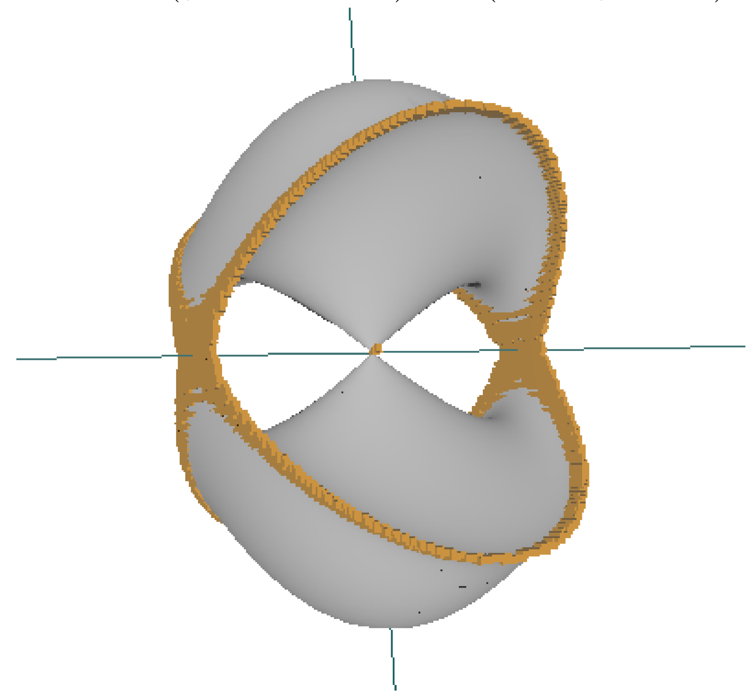
62.								
36.	24.							
22.	14.	9.4						
13.	7.7	4.9	3.4					
7.8	4.2	2.4	1.5	0.95				
4.6	2.2	1.0	0.39	0.06	-0.12			
3.0	1.2	0.33	-0.13	-0.38	-0.52	-0.58		
2.4	0.9	0.16	-0.27	-0.52	-0.66	-0.74	-0.78	
2.6	1.2	0.36	-0.13	-0.45	-0.64	-0.76	-0.83	-0.86

Triangular array of Clown Smile coefficients.

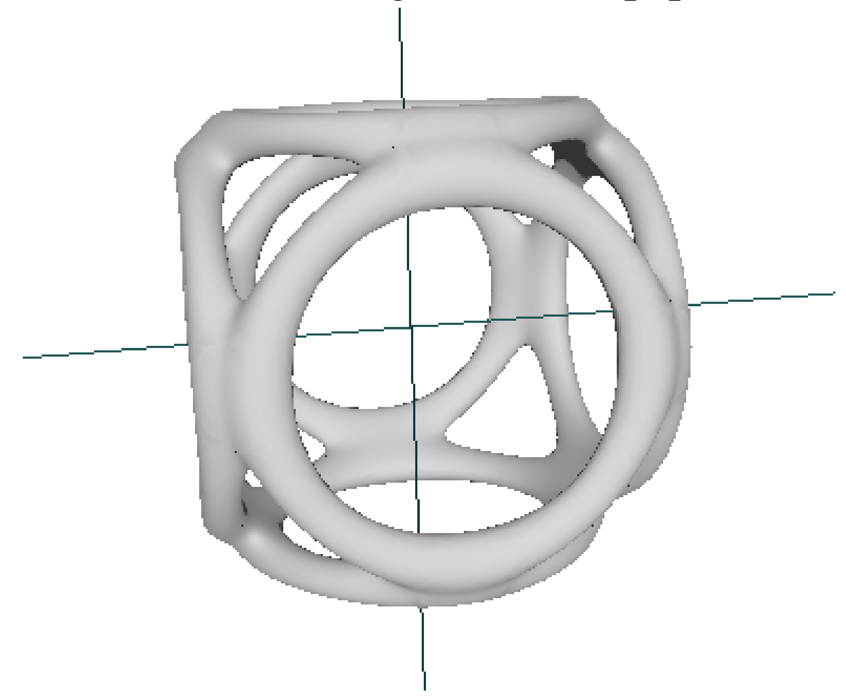
- Observation of coefficients in non-converging regions of Clown Smile suggest additional single-sheet condition for curves:
  - Multiple mixed layers of coefficients, each of which is one sign change and mixed layers are adjacent.
  - Zeros on one sign change layers must be properly ordered
- Surfaces have similar conditions, but heuristics required to make reasonable

Examples

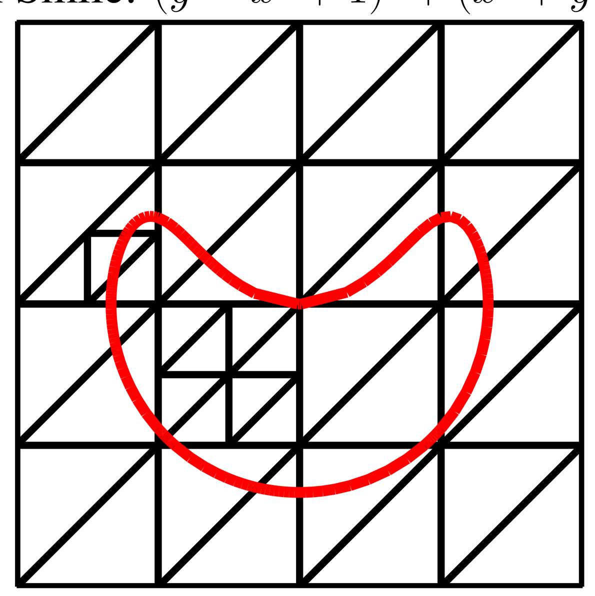
Crixxi: $(y^2 + z^2 - 1)^2 + (x^2 + y^2 - 1)^3$



Decocube: degree 12; see paper



Clown Smile: $(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1$



(3414 triangles reduced to 54)

Cube: $x^6 + y^6 + z^6 - 1 = 0$